(ii) with direction in crystal lattice.

See sheet for free particle / interacting particle $\Big\}$ ϵ vs k graphs.

(2). Many body interactions.

Not all effects of interactions can be treated through single particle approx.

Most important of many body interactions is electron – phonon – electron interaction

Causes

(i) Superconductivity — see later

(ii) most of increase in γ from $C_V = \gamma T$ relation (Guenault p 172)

Statistical Mechanics Theory

N identical particles
Volume V
Thermal equilibrium at temperature T
Weak interactions

Particles distinguishable by position in solid

$$n_j = \frac{N}{Z} \exp\left(-\epsilon_j/kT\right)$$

$$Z = \sum_j \exp\left(-\epsilon_j/kT\right)$$

basic theory

Excitations in localised atoms in solids

Gas particles in macroscopic box. Particles indistinguishable

occupation number $f \gg 1$

Quantum region
Behaviour depends whether $\psi(1,2)$ symmetric or antisymm under particle exchange

$\psi(1,2)$ antisymm

Fermi Dirac statistics

$$f(\epsilon) = \frac{1}{\exp\left((\epsilon-\mu)/kT\right)+1}$$

μ = fermi energy

basic theory

Conduction electrons
Liquid He^3
He^3/He^4 liquid mixtures

$\psi(1,2)$ symmetric

Bose-Einstein statistics

$$f(\epsilon) = \frac{1}{B\exp(\epsilon/kT)-1}$$

basic theory

Radiation in enclosure
Phonons in solids
Superconductors
Liquid He^4

occupation number $f \ll 1$

Classical region
Maxwell Boltzmann statistics

$$f(\epsilon) = \frac{N}{Z}\exp(-\epsilon/kT)$$

$$Z = V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

basic theo[ry]

normal gases

Low temperature Physics.

Superfluidity

Superconductivity

attaining low temperatures

$\boxed{He}$

$\boxed{He}$

liquid

gas

classical gas ✓

liquid He^3

quantum gas

Fermi-Dirac

liquid He^4

quantum gas

Bose-Einstein.

Liquid Helium.

Contrasting properties of liquid He^3 and liquid He^4 illustrates different quantum mechanical behaviour of fermions and bosons.

Liquid He^3

Normal liquid He^3 temp range

$$0.003\,K < T < 3.2\,K$$

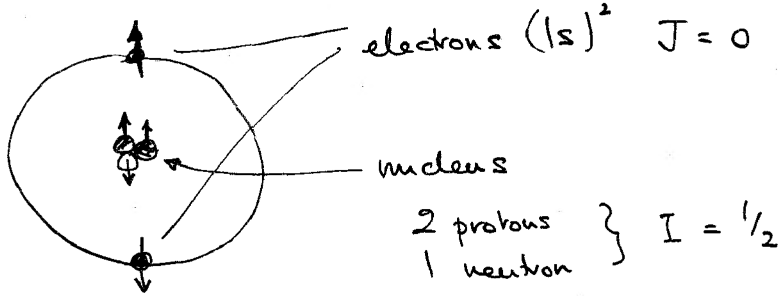
3.2 K is boiling point

Superfluid He^3 temp range $T < 0.003\,K$

$$T < (3\,mK).$$

Treat superfluid later.

He^3 atom is fermion



Total atom $\underline{F} = \underline{I} + \underline{J}$

$\underline{F} = \frac{1}{2} + 0 = \frac{1}{2}$

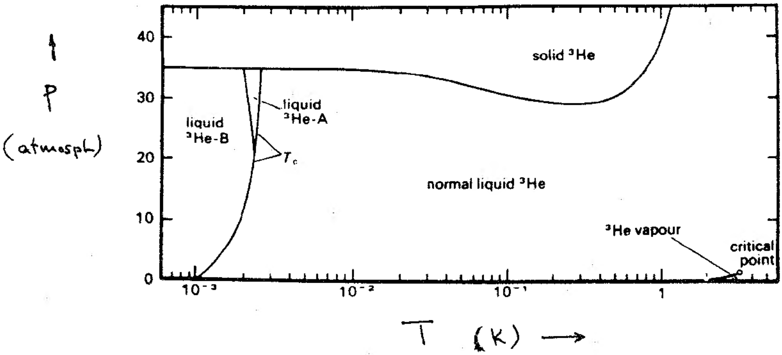
Thus atom a fermion.

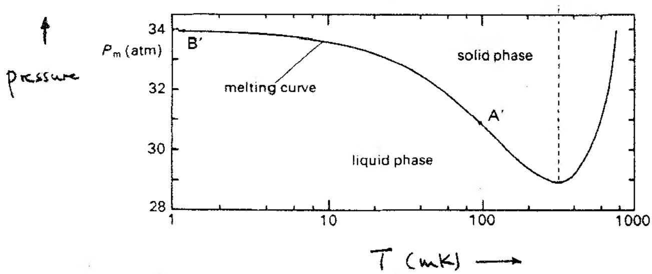
How well does Fermi Dirac gas describe He^3 behaviour?

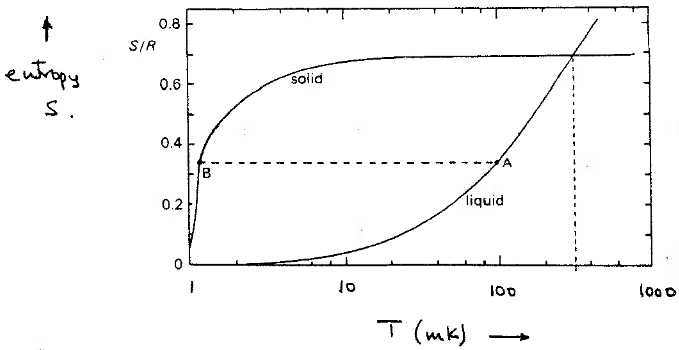
Facts.

Phase diagram (see sheet) is a map that shows state of He^3 under conditions of pressure (p) and temperature (T).

He 3. Phase diagrams.







Unusual features of phase diagram

(1). Under atmospheric pressure He^3 stays liquid to $T = 0$.

Have to apply pressure to solidify it.

(2). For temperature range $T < 0.32K$

Solid – liquid boundary has $\frac{dp}{dT}$ –ve

Discuss these points

(1) Liquid to $T = 0$.

True for liquid He^3 and liquid He^4.

Thus not due to fermion nature.

Interaction between atoms – van der Waals forces (very weak).

To form solid He

Van der Waals binding forces must

overcome

Zero Point Energy (ZPE) – vibration of atom about its site – $\frac{1}{2}h\nu$ / atom due to Uncertainty Principle.

At normal pressure

$$Z.P.E > V.d.W \text{ binding energy}$$

Thus He remains liquid.

2. Negative slope $\left(\frac{dp}{dT}\right)$ in $3mK < T < 0.32K$

Clausius – Clapeyron equation

$$\left(\frac{dp}{dT}\right) = \frac{S_l - S_s}{V_l - V_s}$$

where S_l, S_s — entropy of liquid, solid

V_l, V_s — volume of liquid, solid

for given quantity of He^3.

In Most substances.

$S_l > S_s$ — liquid more disordered than solid

$V_l > V_s$ — solid has higher density than liquid.

Can get $\left(\frac{dp}{dT}\right)$ — ve

either

(i) If $S_l > S_s$ but $V_l < V_s$ (water)

or

(ii) If $S_l < S_s$ and $V_l > V_s$ (liquid He^3)

Grapsh S vs T for liquid He^3 / solid He^3 – see sheet.

In temperature range $3mK < T < 0.32K$

entropy S dominated by disorder of nuclear spin.

In solid He^3 entropy S well described by N localised spin $\frac{1}{2}$ particles.

having 2 energy states separated by energy ϵ

Recall

$$S_s = Nk\left\{\ln\left[1+\exp\left(-\frac{\theta}{T}\right)\right] + \frac{\left(\frac{\theta}{T}\right)\exp\left(-\frac{\theta}{T}\right)}{\left[1+\exp\left(-\frac{\theta}{T}\right)\right]}\right\}$$

where $\theta = \epsilon/k \simeq 2mK$

For $T \gg \theta$

$$S_s \longrightarrow Nk\ln 2$$

Liquid He^3

$$S_\ell = \int_0^T \frac{C_v}{T} dT$$

Fermi Dirac gas $\quad C_v = \gamma T$

Then $\quad S_\ell = \int_0^T \frac{\gamma T}{T} dT = \gamma T$

For $\quad 3\,mK < T < 0.32\,K$

See $\quad S_s = Nk\ln 2 > S_\ell = \gamma T$

Reason.

Solid – distinguishable particles

Boltzmann distribution of spin $\uparrow$ and $\downarrow$

Liquid – indistinguishable particles

Fermi-Dirac distribution – at

low temp all states up to $\epsilon = \mu$

filled with $\uparrow$ and $\downarrow$ – no spin disorder

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- Only particles with energy $\epsilon = \mu - kT$ can flip spin.
- This small number of particles gives

$$S = \gamma T.$$

Practical use of difference of solid and liquid entropy — Pomeranchuk cooling discussed later.

Properties of Liquid He^3

Graphs of

C_v versus T

Thermal conductivity K vs T

Viscosity η vs T

Magnetic susceptibility χ vs T

See sheet.

Comparison expt to Fermi-Dirac gas prediction

(i) Heat capacity

F. D gas

$$C_V = \gamma T = \frac{\pi^2}{3} k^2 T g(\mu)$$

Expt shows reasonable approx to

$C_V \propto T$ for $3\,mK < T < 0.2\,K$

Deviation at higher temperature

(ii) Thermal conductivity K

F. D gas predicts $K \propto \frac{1}{T}$

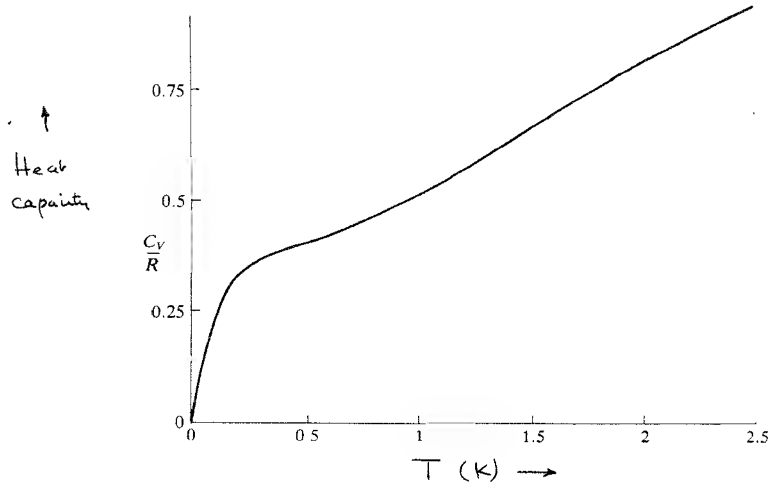
Some agreement in range $3\,mK < T < 0.1\,K$

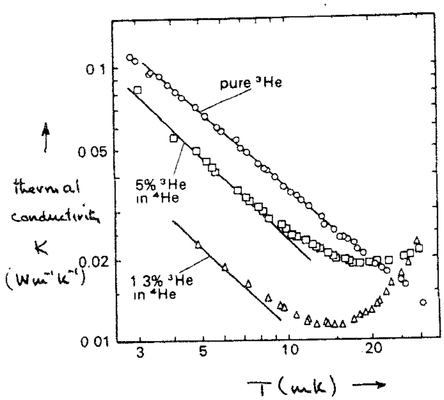
(iii) Viscosity η

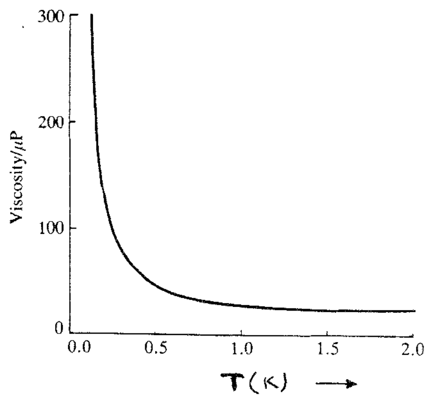
F. D gas predicts $\eta \propto \frac{1}{T^2}$

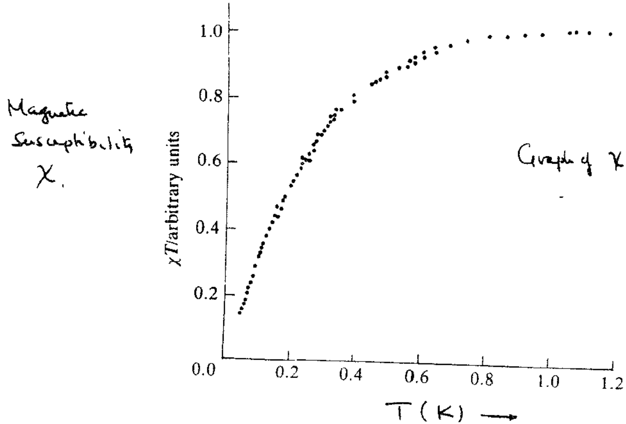
Reasonable agreement with expt up to 2.0 K.

Liquid He^3. Graphs.









Graph of χT vs T.

(141)

Magnetic susceptibility χ.

F.D gas predicts $\chi = \mu_0 \mu^2 g(\epsilon_F)$

independent of temp.

Graph shows χT vs T — straight line

or χ independent of T in range

$3mK < T < 0.4K$.

—

Summary

In range $3mK < T < 0.2K$

F.D gas gives reasonable agreement with experimental graphs.

Quantitative comparison.

Table of coefficients — see sheet.

See quantitative values of coefficients predicted by F.D gas smaller than expt by up to factor 10.

Reason for F.D gas inaccuracy —

- effect of interatomic interactions.

Theory to take account of interactions

Landau theory

Keep single particle approach

Change ϵ vs k relation

— changes energy states — see sheet.

Form of quasiparticle theory where

quasiparticles $\equiv$ particles + averaged effect of interactions

have effective mass m^* ranging

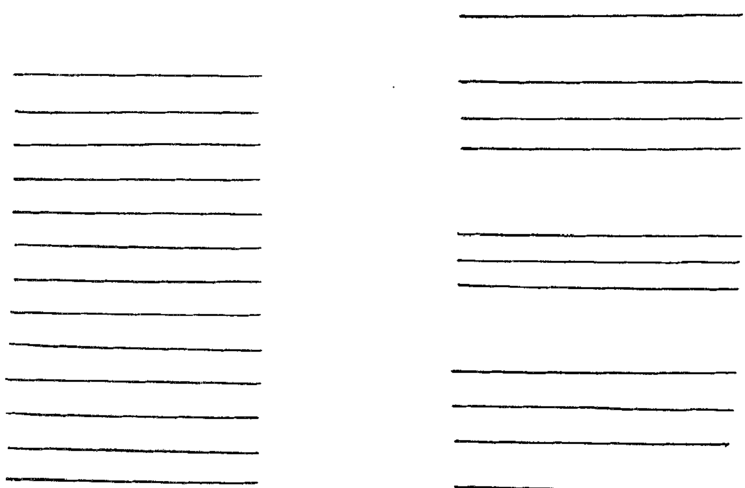
3 – 6 times actual mass.

m^* depends on quantity measured.

Comparison of Liquid He^3 and predictions of Fermi Dirac gas.

QUANTITY	Expt.	Fermi gas prediction	Ratio
Heat capacity / mol. C_V	$2.78\,RT$	RT	2.78
Velocity of sound c (ms^{-1})	183	95	1.92
Magnetic susceptibility χ	$3.3 \times 10^{+38} \beta^2$	$3.61 \times 10^{37} \beta^2$	9.1

β = magnetic moment of He^3 nucleus.

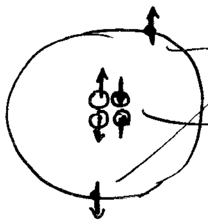


Fermi gas.

Fermi liquid.
includes interactions

Liquid He^4

He^4 atom is a boson



Total atom $\underline{F} = \underline{I} + \underline{J}$

$F = 0 + 0 = 0$ – boson.

Phase diagram – see sheet.

Points.

(i) He^4 remains liquid to $T \to 0$ under normal pressure (needs 25 atmosph to solidify)

Same reason as He^3 – van der Waals interaction's between atoms weaker than Zero point energy

(ii) Flat solid / liquid transition line for $0 < T < 2K$.

No nuclear spin disorder ($I = 0$) – little entropy S in liquid (S_ℓ) or solid (S_s)

Hence Clausius Clapeyron

$$\frac{dp}{dT} = \frac{S_\ell - S_s}{V_\ell - V_s} \simeq 0$$

(iii) Main feature

Phase transition liquid He II / liquid He I at 2.17 K (and $p = 1$ atmosph).

Graphs of S vs T } across phase
C_V vs T } boundary – see sheet.

Graphs show

(i) Steep increase in entropy on transition liq He II $\longrightarrow$ Liq He I.

(ii) From $C_V = T\dfrac{dS}{dT}$ get λ shape

– called λ transition

Properties of liquids

Liquid He I — ordinary liquid.

Liquid He II — unique behaviour

— enormous number of expts done

— look at main features.

Liquid He II

Properties.

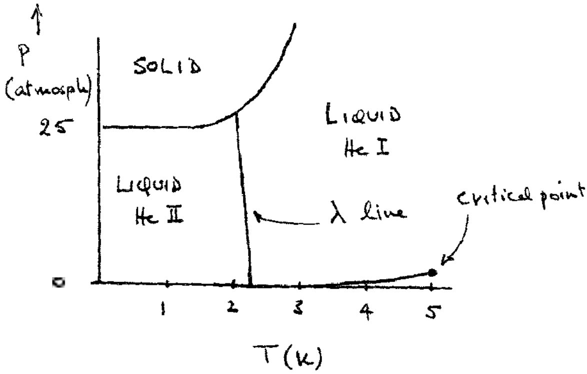
1. Viscosity — value depends on how it is measured. — see sheet.

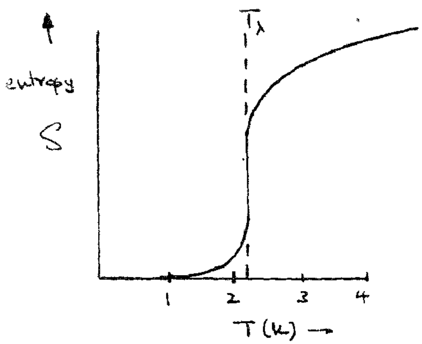
(a) Damping oscillating discs $\eta \sim 10^{-5}$ poise decreases with decreasing T.

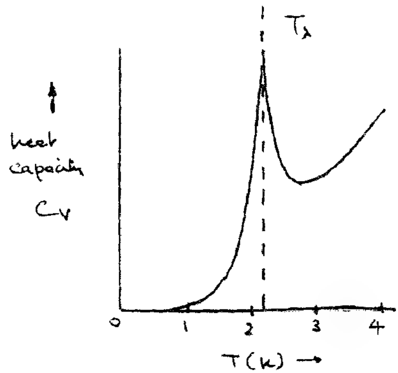
(b) flow through extremely fine capillary $\eta < 10^{-11}$ poise — behaves as if some proportion of liquid had zero viscosity — can flow freely through narrowest channels.

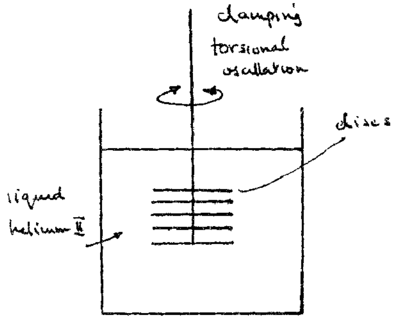
Liquid He^4.

Phase diagram.

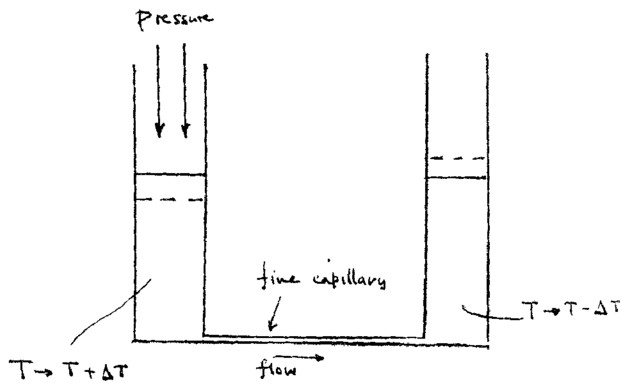








Viscosity $\eta \approx 10^{-5}$ poise



Viscosity $\eta < 10^{-11}$ poise

- vessel empties by siphon flow through film of liquid on vessel walls.

- reservoir of outgoing flow gets warmer
- reservoir of incoming flow gets colder.

(2) Pressure / temperature relation.

Apparatus – see sheet.

Liquid He II in enclosure slightly heated – flow of non viscous component occurs through capillary causing rise of level of liquid in enclosure.

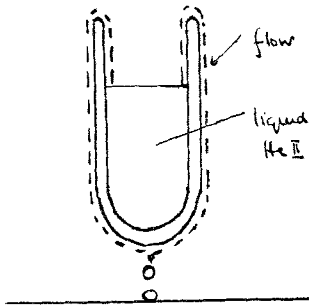
Fountain expt – same effect.

(3) Sound Wave Propagation

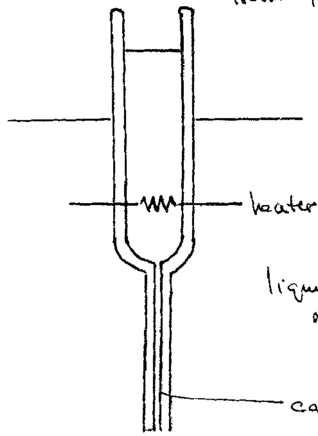
Sound waves – alternating regions of high and low atomic density can pass through liquid He II.

Liquid He II.

Flow thru' surface film.



Pressure/temp relation.



Landau Two Fluid Model.

1. He II behaves as if it consists of two separate fluids – a normal fluid and a superfluid component.

2. The two fluids interpenetrate freely – passing thru' each other without interaction

3. The total density of the liquid is made up of the sum of the densities (number or mass) of the two components

$$\rho\,(\text{total}) = \rho_n + \rho_s \qquad \left.\begin{array}{l} n = \text{normal} \\ s = \text{superfluid} \end{array}\right\} \text{components.}$$

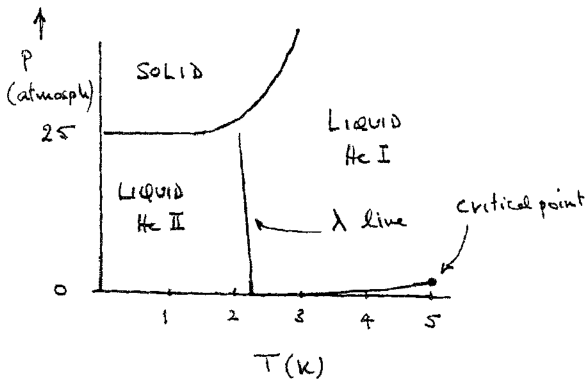
Find $\rho_n \to 0$ as $T \to 0$ $\qquad \rho_s \to 0$ as $T \to T_\lambda$

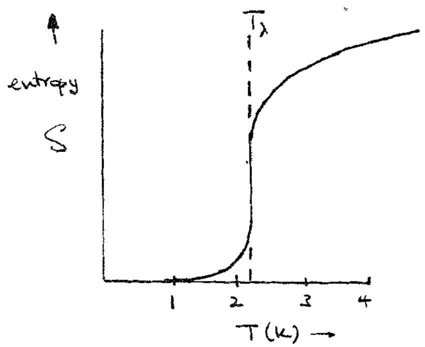
4. Superfluid component carries no entropy and experiences no resistance to flow – its $\eta = 0$ and no turbulence can be created in it.

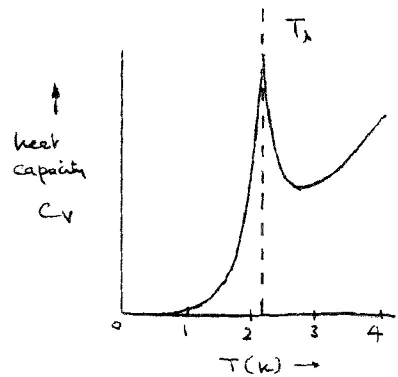
5. Normal fluid carries all entropy S and possesses finite viscosity η.

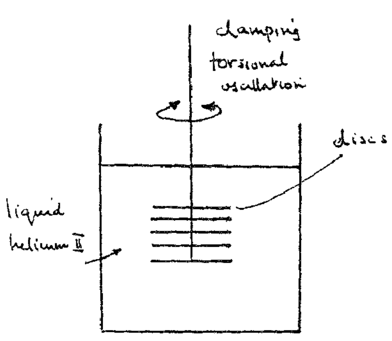
Liquid He^4.

Phase diagram.

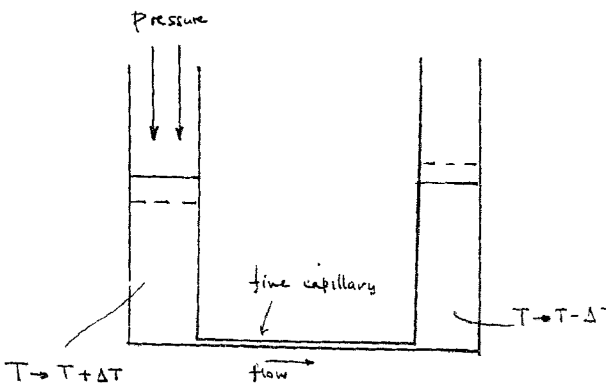








Viscosity $\eta \approx 10^{-5}$ poise



Viscosity $\eta < 10^{-11}$ poise

Other waves – generated by electric heater run at low frequency a.c. sends waves of higher / lower temperature moving thru' liquid He II – called Second Sound.

Other waves detected

Third sound – waves in films

Fourth sound – waves in capillaries

Explanations.

Landau Two Fluid Model – see sheet.

Application to experiments.

1. Viscosity

Damping oscillating discs. – normal component provides damping

Andronikashvili's expt – gives ratio normal / superfluid as function of temp T

see sheet

Flow through capillary – only superfluid flows
– as this component effectively at 0 K
– higher concentration of superfluid cools
– lower " " " warms

(2) Pressure / temperature relation

Warm He II in enclosure – to keep equlibrium ratio of superfluid / normal components, superfluid flows from reservoir to enclosure thru' capillary – causes rise in liquid.

Same with 'fountain expt'.

(3) Sound Waves.

Normal sound – propagation of regions of higher and lower total (superfluid + normal) atomic density.

Second Sound – propagation of regions of

cooler

(high superfluid – low normal) and

(low superfluid – high normal) densities.

warmer

– seen as temperature change wave.

Graphs of velocities of First Sound (u_1)

and Second Sound (u_2) vs T – see sheet.

Q. Can we identify the Two Fluid Model

with Bose - Einstein theory as

Superfluid component $\equiv$ Bose Einstein

ground state condensate

Normal component – Bose-Einstein excited

states thermal population

where $T_B = \lambda$ transition temperature ?

A Broadly - yes but Bose Einstein theory doesn't explain all points.

Points in favour

(i) B-E condensation explains why there are 2 components

(ii) B-E condensate (Superfluid) predicted to have no entropy - as if at 0 K. Thus increasing concentration of this component cools liquid.

(iii) Using experimental value for $\left(\frac{N}{V}\right)$ for He^4 predicts $T_B = 3.1\,K$

close to experimental $T_\lambda = 2.17\,K$